

Limits on defects formation and hybrid inflationary models with three-year WMAP observations

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Abstract. We confront the predicted effects of hybrid inflationary models on the Cosmic Microwave Background (CMB) with three years of *Wilkinson Microwave Anisotropy Probe* (WMAP) observations. Using model selection, we compare the ability of a simple flat power-law Λ CDM model to describe the data to that of hybrid inflationary models involving global or local cosmic strings, or global textures. We find that it is statistically impossible to distinguish between these models: they all give a similar description of the data, the maximum ratio of the various Bayesian evidences involved being never higher than $e^{0.1 \pm 0.5}$. We then derive the maximum contribution that topological defects can make to the CMB, and place an upper bound on the possible value of cosmic strings' tension of $G\mu \leq 2.1 \times 10^{-7}$ (68% confidence limit). Finally, we give the corresponding constraints on the D-term strings' mass scale, as well as limits on the F- and D-term coupling constants (κ and λ) and inflationary scales (M and $\sqrt{\xi}$).

Keywords: CMBR experiments, inflation, physics of the early universe

1. Introduction

Spontaneous symmetry-breaking (SSB) phase transitions were first invoked to explain the evolution of condensed matter systems, such as ferromagnets, as a function of their temperature [1, 2]. It was soon realized that what was true for classical quantum objects and was one of the backbones of the theories of superfluidity and superconductivity [3], could also be applied in the context of quantum field theory.

Kirzhnits [4] and Kirzhnits and Linde [5] first exhibited the analogy between elementary particles' symmetries and these classical systems, arguing that vacuum symmetries that are broken today can be restored at high temperatures in the early Universe. This idea, later developed at the same time by Kirzhnits and Linde [6], Weinberg [7] and Dolan and Jackiw [8], led to the concept of a grand unified group of symmetries G that could have broken down into the group of symmetries describing the forces of Nature we know today, namely $SU(3) \times U(1)_{\text{em}}$ [9, 10]. The previous success of Glashow [11] and Weinberg [12] in describing the electromagnetic and weak forces in the context of the single group of symmetry $SU(2) \times U(1)$ gave even more weight to the idea of grand unification.

However, when a spontaneous symmetry-breaking phase transition occurs between a given group of symmetry and another one exhibiting a non-trivial set of degenerate ground states, it is expected that topological defects will form at the border of domains which end up in different minima. Kibble [13, 14] proposed a mechanism explaining the formation of defects as a function of the homotopy groups of the manifold of degenerate vacua, as well as a qualitative description of their cosmological evolutions.

It was soon realized that the production of monopoles, point-like defects, as well as domain walls, two-dimensional defects, had to be highly constrained so as to be consistent with observations [15, 16]. However, before observations of the Cosmic Microwave Background (CMB) become available on a wide range of angular scales, cosmic strings (unidimensional defects) were weakly constrained by observations. Moreover, it was proposed that they could produce the primordial perturbations needed in the early Universe to form the large-scale structures we observe today. In this scenario, cosmic strings seed the formation of structures by gravitational clustering [17, 18], achieving a similar result as inflation as far as structure formation is concerned.

But if inflation and the existence of topological defects without inflation are both compelling mechanisms to explain structure formation, they lead to dramatically different predictions of the CMB anisotropies and polarization angular power spectra: inflation predicts the existence of multiple peaks in all of these spectra [19, 20], whereas cosmic defects always lead to spectra with a single bump [21, 22, 23, 24, 25].

Therefore, as more and more precise measurements of the CMB were made, it became possible to test the cosmic strings scenario against inflation. These observations now clearly show multiple peaks in the angular power spectrum of the CMB anisotropies [26, 27, 28, 29, 30, 31], as well as in the corresponding polarized signal [32, 33, 34, 35, 36]. Cosmic defects alone are therefore unable to fit the

CMB temperature angular power spectrum or to give a good description of the observed polarization.

But hybrid inflation [37, 38] and supersymmetric hybrid inflation [39, 40, 41], which predict the coexistence of topological defects with an inflationary phase in their standard version [38, 42], have been shown to be consistent with CMB observations as soon as the first observation of a succession of acoustic peaks was made in the CMB temperature anisotropies power spectrum [43]. Later studies have explored both cosmic strings and various kind of cosmologically acceptable defects in light of recent CMB measurements [44, 45, 46, 47].

In this paper, we study what the newly released WMAP results can teach us about hybrid inflation[‡]. We first look at whether introducing topological defects to explain WMAP observations is justified in a Bayesian analysis. If it is indeed always possible to add a contribution of defects and fit it to the data, this does not tell us anything about whether adding this extra parameter gives a better description of the observations, and this is what we should first check, using the Bayesian evidence as a selection tool [49, 50]. We then constrain hybrid models involving various kind of defects using a full Markov Chain Monte Carlo analysis. Finally, we translate these constraints into bounds on the free parameters of D- and F-term inflationary models.

2. Model selection

To constrain the values of cosmological parameters, one must first choose a set of parameters that will then be fitted to the data. To do so, a solution is to choose a model ad hoc, with the criteria that it must both be theoretically attractive and give a good description of the data. It turns out that this selection has no unique answer. It is, for example, possible to fit WMAP data using a running or a non-running spectral index, both giving a comparable description of the CMB anisotropies [51]. Therefore, one needs an additional tool to be able to discriminate between two models of different dimensions.

Model selection makes such a comparison possible. This technique rests on the observation that adding a parameter to a given model and fitting this new model to the data almost always gives a better fit unless a strong prior is added on the value of this extra parameter [52]. In other words, a model should be penalized compared to another one if it requires more parameters to get a marginally better fit, as should a model requiring fewer parameters but leading to a less satisfying fit.

In the context of Bayesian analysis, one of the best way of *grading* a model is to

[‡] Unless otherwise stated, we study the case where no *a priori* theoretical constraints are taken into account. For example, we do *not* include the fact that the spectral index n_s can usually not be smaller than 0.98. This allows our constraints to stay valid even if $n_s \leq 0.98$, which can be the case when the radiative corrections arising from the introduction of a non-minimal Kähler potential are taken into account. These two points are discussed, for example, in [48].

look at its Bayesian evidence [53]

$$E = \int \mathcal{L}(\mathbf{s}) \Pr(\mathbf{s}) d\mathbf{s} \quad (1)$$

where \mathbf{s} is a given set of parameters, $\mathcal{L}(\mathbf{s})$ the likelihood of this model in light of the data and $\Pr(\mathbf{s})$ the priors on the chosen parameters. A model is favoured by the data compared to another one if its evidence is higher. However, for this to be significant, the difference $\Delta \ln E$ between two models should be at least of order 2.3, in which case the model of higher evidence is about ten times more likely to be a better description of the observations than the other one [49]. But we would need a difference of 5 to 6 to claim that the data decisively reject one of the models compared to the other one.

We consider two models: a power-law flat Λ CDM model, characterized by six parameters, and a hybrid model in which the power spectrum of the CMB anisotropies is

$$C_\ell = (1 - \alpha) C_\ell^{\Lambda\text{CDM}} + \alpha C_\ell^{\text{TD}} \quad (2)$$

where α , the contribution of topological defects (TD), is a seventh parameter and we compute the evidence for each model by thermodynamic integration. In practice, this means that E is calculated for each model by running Markov chains in which the acceptance criterion is given by the likelihood to the power θ (instead of 1), where θ is a parameter slowly varying from 1 to 0 as the chain runs. Following [53], we let θ continuously vary with the number of steps n in the Markov chain. At the n th step, $\theta_n = (1 - \xi)^n$, where ξ is a constant empirically set to 5×10^{-5} to optimize the speed and accuracy of the determination of E . The chain is stopped after n_{max} steps, where the step $n_{\text{max}} + 1$ would change $\ln E$, defined by

$$\ln E = \sum_{n=0}^{n_{\text{max}}} [\ln \mathcal{L}(\mathbf{s})]_n \xi (1 - \xi)^n \quad (3)$$

by less than 10^{-3} . As we did in [46], we generate the adiabatic spectrum at each step of a Markov chain with CMBwarp [54]. The results are similar to what we get with CAMB in the range of multipoles and cosmological parameters we consider. The situation is much simpler for topological defects, as only the overall normalization of the angular power spectra they induce can change between two consecutive steps. Therefore, we only need to generate the corresponding spectra once, before running the chains, and choose an initial normalization.

As it is interesting to be able to compare how the constraints have evolved between the two WMAP releases, we choose the normalization used in [46], for which the power in the angular power spectrum is kept to the value predicted by the *first-year* (and not the three-year) WMAP best fit with six parameters, independently of the value taken by α . The value of the latter is consequently dependent on this normalization, which one should take into account when comparing different works. We then choose the shapes of the spectra induced by topological defects and their relative amplitudes in [45], which considers all cosmologically motivated defects along with the differences in the predictions of different models of the same kind of defects.

Table 1. Upper limits on the contribution of defects to the CMB angular power spectrum (%), and corresponding strings' tensions ($G\mu$) for three different models. The tension given for global strings is the one a local string with the same spectrum would have. In a given cell, the first (second) number is the 68% (95%) confidence level upper bound.

Defects		Upper bound	$G\mu \times 10^7$
Global strings	[21]	13% – 18%	2.4 – 2.8
Local strings	[25]	7% – 11%	2.1 – 2.6
Local strings	[22]	5% – 7%	2.1 – 2.5

We explore the parameter space using flat priors on all our cosmological parameters such that $(\omega_b, \omega_m) \in [0, 1]^2$, $h \in [0.5, 1.5]$, $A \in [0.5, 2.5]$, $\tau \in [0, 0.3]$, $\alpha \in [0, 1]$ and $n_s \in [0, 2]$. We want to get an evaluation of $\ln E$ to better than 1%, which requires us to run at least 25 different Markov chains satisfying the stopping criterion for each model.

We find that it is statistically impossible to distinguish between a flat power-law Λ CDM model and a hybrid model from the point of view of model selection. The ratio between the evidence of the adiabatic model and the evidence of all the other models we tested is indeed always less than $e^{0.1 \pm 0.5}$.

In other words, a Λ CDM model, which is a particular case of any hybrid model, is as good a description of the data as a hybrid model with topological defects, and there is no obvious need to introduce the latter. The data being indifferent to the set of parameters used, the next logical step is to determine what values of the cosmological parameters of a hybrid model can fit the data, and what are the constraints on the added contribution of topological defects.

3. Best fit hybrid models

The method used to find the best fit hybrid models is entirely similar to the one developed in the previous section, except that the acceptance criterion is governed by the likelihood of the model, instead of a varying power of the latter. This analysis therefore follows the algorithm used in [55] for a flat power-law Λ CDM model.

It is also much less computationally expensive than the model selection analysis. Performing the tests described in [46], we indeed find that running eight chains of 2×10^5 steps each is sufficient to get a value of the Gelman and Rubin convergence diagnostic down to 1.1 [56]. Getting the best fit models is therefore about four times faster than checking if the data favours the addition of an extra parameter.

We give the 68% and 95% confidence level upper bounds on the contribution of one model of global cosmic strings and two models of local cosmic strings in table 1. In addition to these models, we also considered the model of [23], for which we get similar results. Finally, global and non-topological textures produce spectra indistinguishable from the model of global strings we use [21], so that their contributions are subject to

the same upper bound.

We find that the results are *weakly* dependent on the model of defects we consider. Even when we use the spectrum induced by global strings to get a constraint on $G\mu$, the corresponding bound differs from the value derived with local strings by only 8% at 3σ . In each case, the best fit deviates from 0% at 1σ , as one would expect when an extra parameter is added.

Combining these different results, we can choose a conservative upper bound (68% CL) of $G\mu \leq 2.1 \times 10^{-7}$, corresponding to a contribution of 6% of local strings to the CMB temperature angular power spectrum. This is all the more reasonable that recent measurements of the third acoustic peak (weakly constrained by the new WMAP results [26]) by VSA [27], BOOMERanG [28], and CBI [31] suggest that it lies close to the predicted Λ CDM spectrum [51]. Therefore, as defects produce less power at the scale of the third acoustic peak than at the one corresponding to the second peak [45], the contribution of defects should go down when these data are taken into account.

4. Constraints on hybrid inflationary models

Standard hybrid inflation can avoid the formation of various kind of defects but cosmic strings are particularly unavoidable [57, 58] in the context of SUSY GUTs§. A constraint on the existence of cosmic strings is therefore a direct cosmological constraint on these unification theories.

GUT strings formed at the end of F-term inflation do not saturate the Bogomolny bound, whereas strings formed in D-term inflation do [59]. The energy scale of the SSB leading to the production of the corresponding defects is therefore dependent on what scenario one considers. However, this can be understood with a unified formalism in which the mass scale M of a theory is given by

$$M = M_{\text{Pl}} \sqrt{\frac{G\mu}{2\pi}} \frac{1}{\sqrt{\epsilon}} \quad (4)$$

where M_{Pl} is the Planck mass and ϵ is a dimensionless parameter whose value is in $[0, 1]$ and depends on the type of inflation one considers. Using the constraint on $G\mu$ that we previously derived, it is then possible to find an upper bound on M as a function of ϵ for the theories mentioned above.

In the case of D-term inflation, in which strings satisfy the Bogomolny limit, $\epsilon = 1$. In GUT F-term inflation, the SUSY superpotential W_{F} can be written as

$$W_{\text{F}} = \kappa S (\phi_+ \phi_- - M^2) \quad (5)$$

where S is the slow-rolling field of the theory, ϕ_+ and ϕ_- are chiral GUT Higgs superfields, and κ is a coupling constant. ϵ is then a function of κ whose variation obeys a different law for $\kappa \leq 10^{-1}$ and $\kappa \geq 10^{-1}$. In the latter case, ϵ is well approximated

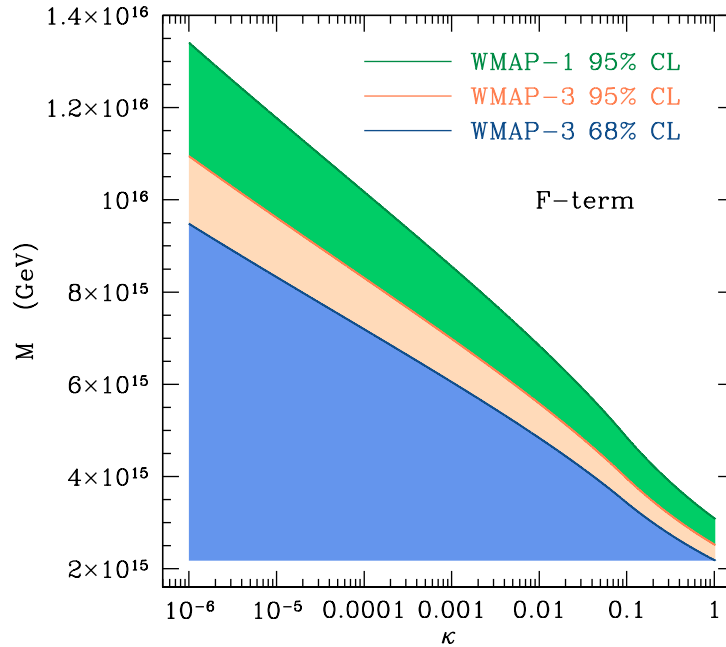
§ Note that this is not the case for all kinds of hybrid inflationary models, as some of them, e.g. smooth and shifted hybrid inflations, do *not* lead to the formation of defects [60].

by $\epsilon = 1.04 \times \kappa^{0.39}$ [59]. Moreover, when $\kappa \rightarrow 0$, $\epsilon \rightarrow 2.4/\ln(2/\kappa^2)$ [61]. Interpolating between those two regimes, one can thus take ϵ as

$$\epsilon = \begin{cases} 1.04 \times \kappa^{0.39} & \text{if } \kappa \geq 10^{-1} \\ 2.4 [\ln(2/\kappa^2)]^{-1} - 2.93 \times 10^{-2} & \text{if } \kappa \leq 10^{-1} \end{cases} \quad (6)$$

where this last constant is chosen so that ϵ be a continuous function of κ . Due to this dependence of ϵ upon κ in GUT F-term inflation, our limits on $G\mu$ translate into a constraint on what area of the (M, κ) -space is allowed by three years of WMAP data. These results are shown in figure 1.

Figure 1. Constraints on the (M, κ) -space of F-term hybrid inflation from one and three years of WMAP results. The blue (respectively orange) curve corresponds to the 68% (resp. 95%) confidence limit (CL) upper bound using three years of WMAP data. The blue-shaded area is the region of the parameter space allowed at the 68% CL. The orange-shaded area is the additional allowed region if one considers the 95% CL. The green curve corresponds to the average 95% CL upper bound using one year of WMAP data as derived from [44, 45, 46]. The green-shaded area is therefore the region of parameter space allowed by one year, but excluded by three years, of WMAP data.



In the case of D-term inflation, as $\epsilon = 1$, it is possible to get a direct constraint on the mass scale of the theory, using equation (4). Moreover, the SUSY superpotential W_D of the theory can be written

$$W_D = \lambda S \phi_+ \phi_- \quad (7)$$

where we use notations similar to the ones used for W_F , and λ is the superpotential coupling constant. For this kind of hybrid inflationary model, [62] showed that it is possible to find a relationship between λ and the strings' contribution to the CMB

angular power spectrum, as well as between λ and the Fayet-Iliopoulos term ξ , which are two of the three free parameters of D-term inflation. However, the relationship between λ and the strings' contribution is dependent on the third free parameter of the theory, namely the gauge coupling constant g . Though the effect of g on the shape of this function is shown to be pretty dramatic in [62], it is hard to set a stringent constraint on the gauge coupling constant from a given contribution of D-term strings, unless the latter is very low (a few hundredth of a percent) or very high (close to 100%). From our constraints, we can only feel confident that the maximum value of g allowed by three years of WMAP data is of the order of 10^{-2} , leading to

$$\lambda \leq 2.3 \times 10^{-5} \quad \text{and} \quad \sqrt{\xi} \leq 2.0 \times 10^{15} \text{ GeV}. \quad (8)$$

This last parameter can also be considered as the inflationary scale of the theory. We summarize the constraints derived in this section for D-term inflation and for one and three years of WMAP observations in table 2.

Table 2. Upper bounds on the D-term strings (M_s) mass scale and the D-term coupling constant (λ) and inflationary scale ($\sqrt{\xi}$) from one and three years of WMAP observations.

Parameters	1-year	3-year
$M_s \times 10^{-15}$ (GeV)	2.9	2.2
$\lambda \times 10^5$	4.5	2.3
$\sqrt{\xi} \times 10^{-15}$ (GeV)	2.6	2.0

5. Conclusions

Using three years of WMAP data, we showed that it is not possible to exclude hybrid inflationary models by a statistical analysis. The data does not prefer a Λ CDM description to a model involving global strings, global textures or local strings, even if we take into account the dimensions of the models we compare, as was the goal of our model selection analysis. On the other hand, there is no compelling evidence requiring the introduction of cosmic defects in addition to the six parameters of a power-law flat Λ CDM model. Moreover, the WMAP three-year data is precise enough to set stronger constraints on topological defects than what was previously found with one year of WMAP observations.

We find that the tension of cosmic strings must be less than $G\mu = 2.6 \times 10^{-7}$ at 3σ , a more conservative value being $G\mu \leq 2.1 \times 10^{-7}$, which corresponds to the 68% confidence level upper bound^{||}. This limit does *not* depend on the model of local strings we use. Even when we consider that local strings are similar to global strings, the discrepancy is only of order 8% at 3σ . From these constraints, we then derive upper

^{||} After this paper was submitted, [63] reported similar constraints, using a combination of WMAP three-year results and Lyman-alpha forest, galaxy clustering and supernovae constraints.

bounds on the free parameters of D- and F-term inflation, a summary of which is given in figure 1 and table 2.

Hybrid inflation is therefore becoming more and more constrained by CMB measurements, but not yet to a level requiring the introduction of unnatural modifications in the theory. As this scenario does not involve more fine tuning than *classical* inflation, is perfectly consistent with all current observations, and is naturally predicted by SUSY GUTs, it is worth keeping it in mind.

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